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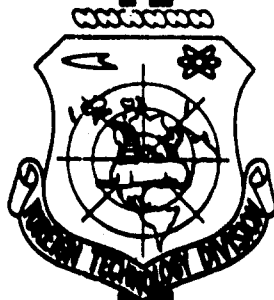
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# TRANSLATION

NEWS OF THE ACADEMY OF SCIENCES OF THE USSR. DEPARTMENT  
OF TECHNICAL SCIENCES, MECHANICS AND MACHINERY  
MANUFACTURE (SELECTED ARTICLES)

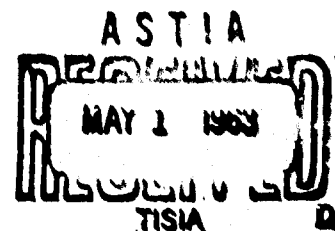
## FOREIGN TECHNOLOGY DIVISION



AIR FORCE SYSTEMS COMMAND

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# UNEDITED ROUGH DRAFT TRANSLATION

NEWS OF THE ACADEMY OF SCIENCES OF THE USSR. DEPARTMENT OF  
TECHNICAL SCIENCES. MECHANICS AND MACHINERY MANUFACTURE  
(SELECTED ARTICLES)

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## TABLE OF CONTENTS

	Page
Effect of Joule Heating on Heat Transmission at the Critical Point, by V. A. Polyanskiy.....	1
Determining the Base Pressure and Base Temperature on the Sudden Expansion of a Sonic or Supersonic Flow, by R. K. Tagirov.....	8

# EFFECT OF JOULE HEATING ON HEAT TRANSMISSION AT THE CRITICAL POINT

by

V. A. Polyanskiy

(Moscow)

The possibility of reducing the heat transmission to the surface of a body moving with hypersonic speed with the aid of a magnetic field created in it was investigated by I. Neuringer and W. McIlroy [1], V. Rossow [2], and R. Meyer [3]. It was shown that it is possible to reduce the heat flows to a body by 20 to 25%, but to accomplish this there are necessary powerful magnetic fields (of the order of 3,000 gauss). However, in these investigations in calculating the heat flow the Joule heat was not taken into consideration. This work was conducted for the purpose of bringing out the influence of the Joule heating on the heat transmission.

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We will consider all the physical characteristics of a fluid (viscosity, thermal and electrical conductivity, etc.) as being constant. We will write the equation of the magnetic hydrodynamics of an incompressible liquid

$$\begin{aligned} \operatorname{div} \mathbf{V} &= 0 \\ (\mathbf{V} \cdot \nabla) \mathbf{V} &= -\frac{1}{\rho} \nabla p + \frac{1}{4\pi\rho} \operatorname{rot} \mathbf{H} \times \mathbf{H} + \nu \Delta \mathbf{V} \\ \operatorname{div} \mathbf{H} &= 0 \\ \operatorname{rot} \mathbf{V} \times \mathbf{H} - \nu_m \operatorname{rot} (\operatorname{rot} \mathbf{H}) &= 0 \\ (\mathbf{V} \cdot \nabla) T &= \kappa \Delta T + \frac{1}{\rho c_p} \mathbf{V} \cdot \nabla p + \Phi + \frac{\nu_m}{4\pi\rho c_p} (\operatorname{rot} \mathbf{H})^2 \end{aligned} \quad (1)$$

Here  $\Phi$  is the dissipative function,  $\nu_m$  is the magnetic viscosity, and  $\kappa$  is the coefficient of heat conductivity

$$\begin{aligned} \Phi &= \mu \left( \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) \frac{\partial u_k}{\partial x_i} \\ \nu_m &= \frac{c^2}{4\pi\sigma}, \quad \kappa = \frac{k}{\rho c_p} \end{aligned} \quad (1a)$$

Besides, we will use Ohm's generalized

$$\mathbf{j} = \sigma (\mathbf{E} + \frac{1}{c} \mathbf{V} \times \mathbf{H}) \quad (2)$$

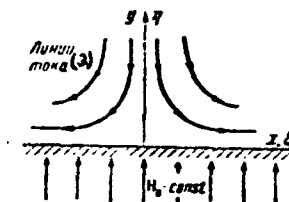


Fig. 1

Key: (a) lines of flux

We will consider the plane flow of a fluid in the vicinity of the critical point. The outer constant magnetic field  $H_0$  we will direct perpendicularly to the surface of the body, and this surface we will assume to be nonconducting (Fig. 1). In the system of equations (1) we will make the usual simplifications of the theory of the boundary layer. For evaluating the magnitude of the magnetic terms one may make use of the simplest single-dimension flow of a fluid along a nonconducting wall in a transverse magnetic field. The integration of the equations of induction

$$H_0 \frac{du^*}{dy} + v_m \frac{d^2 H_x^*}{dy^2} = 0 \quad (2a)$$

with the boundary conditions on the wall  $H_x^* = 0$ ;  $dH_x^*/dy = 0$  gives

$$\frac{H_x^*}{H_0} = -\frac{\delta}{L} R_m \int_0^1 \frac{u^*}{V} d\eta \quad \left( R_m = \frac{VL}{v_m} \right) \quad (2b)$$

Here  $\delta$  is the thickness of the boundary layer,  $L$  is some linear dimension, and  $R_m$  is Reynold's magnetic number. In limiting ourselves to  $R_m < 1$  we get

$$\frac{H_x^*}{H_0} \sim \frac{\delta}{L} \quad (3)$$

From the equation  $\text{div } H^* = 0$ , taking into account the evaluation for  $H_x^*$  there follows

$$\frac{H_y^*}{H_0} \sim 1 \quad (4)$$

We will introduce the dimensionless magnitudes

$$\xi = x \left( \frac{a_0}{v} \right)^{\frac{1}{2}}, \quad \eta = y \left( \frac{a_0}{v} \right)^{\frac{1}{2}}, \quad u = u^* (a_0 v)^{-\frac{1}{2}}, \quad v = v^* (a_0 v)^{-\frac{1}{2}} \quad (5)$$

$$H = \frac{H^*}{H_0}, \quad p = p^* (\rho_0 a_0 v)^{-1}, \quad T = T^* c_p (a_0 v)^{-1}$$

Here  $a_0$  is the parameter with the dimension  $\text{sec}^{-1}$ . Let us introduce

the designations

$$S = \frac{H_0^2}{4\pi\rho_0 a_0 v_m}, \quad \beta = \frac{v_m}{v}, \quad P = \frac{v}{\kappa} \quad (6)$$

The system of equations (1) we will set up in the following fashion:

$$\begin{aligned} u \frac{\partial u}{\partial \xi} + v \frac{\partial u}{\partial \eta} &= -\frac{\partial p}{\partial \xi} + S\beta \left( H_\eta \frac{\partial H_\xi}{\partial \eta} - H_\xi \frac{\partial H_\eta}{\partial \xi} \right) + \Delta u \\ \frac{\partial p}{\partial \eta} &= 0, \quad \frac{\partial}{\partial \eta} (u H_\eta) - \beta \frac{\partial}{\partial \eta} \left( \frac{\partial H_\eta}{\partial \xi} - \frac{\partial H_\xi}{\partial \eta} \right) = 0 \\ \frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} &= 0, \quad \frac{\partial}{\partial \xi} (u H_\eta) - \beta \frac{\partial}{\partial \xi} \left( \frac{\partial H_\eta}{\partial \xi} - \frac{\partial H_\xi}{\partial \eta} \right) = 0 \\ u \frac{\partial T}{\partial \xi} + v \frac{\partial T}{\partial \eta} &= u \frac{\partial p}{\partial \xi} + \frac{1}{P} \frac{\partial^2 T}{\partial \eta^2} + \left( \frac{\partial u}{\partial \eta} \right)^2 + S\beta^2 \left( \frac{\partial H_\eta}{\partial \xi} - \frac{\partial H_\xi}{\partial \eta} \right)^2 \end{aligned} \quad (7)$$

Boundary conditions:

$$\text{at the wall } u^* = v^* = 0, \quad T^* = T_w^*, \quad H_y^* = H_0, \quad H_x^* = 0, \quad p^*(0, 0) = p_0^* \quad (7a)$$

$$\text{at the edge of the boundary layer } u^* = ax, \quad v^* = -ay, \quad T^* = T_\infty^* \quad (7b)$$

In the dimensionless magnitudes the boundary conditions have the form:

$$\begin{aligned} \text{at the wall } u = v = 0, \quad T = T_w^* c_p (a_0 v)^{-1}, \quad p(0, 0) = p_{\infty}^* (p_0 a_0)^{-1} \\ H_x = 0, \quad H_y = 1 \end{aligned} \quad (8)$$

at the edge of the boundary layer

$$u = \frac{a}{a_0} \xi, \quad v = -\frac{a}{a_0} \eta, \quad T = T_\infty^* c_p (a_0 v)^{-1} \quad (8a)$$

Let us introduce the function of the flow  $\psi = \xi f(\eta)$  for the fluid and  $G = \xi g(\eta)$  for the magnetic field. Then

$$u = \xi f'(\eta), \quad v = -f(\eta), \quad H_x = \xi g'(\eta), \quad H_y = -g(\eta) \quad (9)$$

The temperature and pressure we will seek in the form of an analysis by degrees of  $\xi$  (limiting ourselves in this to members of the second order)

$$p = p_0 + G\xi^2 \quad (C = \text{const}), \quad T = T_0 + (T_w - T_0)\theta_1(\eta) + \xi^2\theta_2(\eta) \quad (10)$$

Here  $T_w$  is the dimensionless temperature of the wall, and  $p_0$  and  $T_0$  are the dimensionless parameters of the retardation. The magnitude  $\xi$  in the first degree does not enter here because of the symmetry of the flow. By substituting (9) and (10) in the system (7) we will get a system of ordinary differential equations

$$\begin{aligned} f'^2 - f''f - f'' + S\beta g g'' = -2C \\ 3g'' - f''g - f'g' = 0, \quad \beta g'' - f'g' = 0, \quad \theta_1'' + Pf'\theta_1 = 0 \\ \theta_2'' + Pf'\theta_2 - 2Pf'\theta_1 = -2CPf' - P(f'^2 + S\beta^2 g'^2) \end{aligned} \quad (11)$$

The constant  $C$  we will determine by integrating along the wall the equation of the amount of motion of the nonviscous fluid for the component of velocity  $u$ , and we will get

$$p = - \int_0^\xi \left\{ u(\xi, 0) \frac{\partial u}{\partial \xi} - v(\xi, 0) \frac{\partial u}{\partial \xi} - \right. \quad (12)$$

$$\left. - S\beta \left[ H_y(\xi, 0) \frac{\partial H_x}{\partial \eta} - H_x(\xi, 0) \frac{\partial H_y}{\partial \xi} \right] \right\} d\xi = \text{const} - \frac{1}{2} \xi^2 \frac{a}{a_0} \left( \frac{a}{a_0} + S \right)$$

since in a nonviscous flow at the wall

$$u = \frac{a}{a_0} \xi, \quad v = 0 \quad (12a)$$

The influence of the magnetic field on the distribution of the pressure at the wall near the critical point may be disregarded. Actually, we will

consider the expression for the Lorentz force. From Maxwell's equation

$$j^* = \frac{C}{4\pi} \text{rot } H^* \quad (12b)$$

and Ohm's law (2) it is seen that the intensity of the electrical field  $E$  has a component only along the axis  $z$  (since  $V^*$  and  $H^*$  lie in the plane  $xy$ ).

The equation  $E^* = 0$  shows that 
$$E_z = \beta \left( \frac{\partial H_x}{\partial \xi} - \frac{\partial H_\xi}{\partial \eta} \right) - (uH_x^* - vH_\xi^*) = \text{const} \quad (12c)$$

By making use of the boundary condition at the wall for  $V$  and  $H$  we get  $E_z = \text{const} = 0$

In this way the Lorentz force has the form  $\frac{6}{c} (V^* \wedge H^*) \times H^*$ .

But close to the critical point the directions of the velocity of the flow and the intensity of the magnetic field coincide. Therefore, the magnitude of the Lorentz force here is small differing little from zero. Now by comparing the expression obtained for the distribution at the wall (12) with the analogous expression in the case of nonmagnetic flow

$$p = \text{const} - \frac{1}{2} \xi^2 \quad (13)$$

we get the connection

$$\frac{a}{a_0} = \left( \frac{S^2}{4} + 1 \right)^{\frac{1}{2}} - \frac{S}{2} \quad \left( C - \frac{1}{2} \right) \quad (14)$$

The third equation of the system (11) proves to be an integral of the second equation. Therefore we will not consider one of them.

The validity of the accepted evaluations (3) and (4) is confirmed by the integration of the equations of induction and motion without simplifications (see the equations (25) and (20) of the report [1]).

The final form of the system of equations

$$\begin{aligned} f'' - ff' - f'' + S\beta g g' &= 1 \\ \beta g' - g f' &= 0, \quad \theta_1' + P/\theta_1 = 0 \\ \theta_1' + P/\theta_1 - 2P f' \theta_1 &= P f' - P f'' - S P \beta g g' \end{aligned} \quad (15)$$

The boundary conditions for the functions  $f$ ,  $g$ ,  $\theta_1$  and  $\theta_2$  will be

$$\text{at the wall} \quad f = f' = g' = 0, \quad g = -1, \quad \theta_1 = 1, \quad \theta_2 = 0 \quad (15a)$$

$$\text{at the edge of the boundary layer} \quad f' = \frac{a}{a_0}, \quad \theta_1 = 0, \quad \theta_2 = -\frac{1}{2} \left( \frac{a}{a_0} \right)^2 \quad (16)$$

The conditions for  $\theta_1$  and  $\theta_2$  are obtained from Bernoulli's integral.

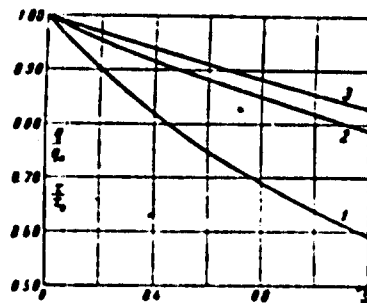


Fig. 2

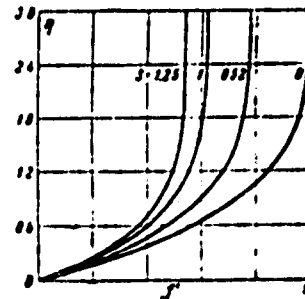


Fig. 3

The system of equations (15) with the boundary conditions (16) was integrated numerically by the Runge-Kutta method on the electronic computation machine "Setun" for different values of the parameter  $S$  and the values  $\beta = 10^6$  and  $P = 0.71$ .

The tangential stress on the wall was computed by the formula

$$\tau = \mu \frac{\partial u}{\partial y} = \rho_0 \nu_0 \frac{f''(0)}{f'(0)} \text{ при } y=0 \quad [\rho \mu = \text{with}] \quad (16a)$$

Consequently

$$\frac{\tau}{\tau_0} = \frac{f''(0)}{[f''(0)]_0} \quad (17)$$

Here zero in the subscript points to the figure for the magnitude in the absence of a field. In Fig. 2 it is seen that with the change in the parameter  $S$  from 0 to 1.5 there occurs a considerable lessening of the tangential friction on the wall (up to 47%, see curve 1). This is the consequence of the lessening of the tangential component of the velocity under the influence of the magnetic field, as a result there is a diminution in the gradient of the velocity on the wall. From Figs. 3 and 4 there is seen the change in the profiles of the components of the velocity  $u$  and  $v$  on the change in the parameter  $S$ .

The magnitude of the heat flow onto the wall is computed in the following way

$$q = k \frac{\partial T}{\partial y} \Big|_{y=0} = - \frac{k_0 \nu_0^{\frac{1}{2}}}{c_p} [(T_0 - T_w) \theta_1'(0) - \frac{1}{2} \theta_2'(0)] \quad (18)$$

The curve 2 in Fig. 2 shows the change in the heat transmission onto the wall with the increase in the parameter  $S$ . The Joule heat in this case is not taken into consideration. The ratio  $q/q_0$  in this case has the form

$$\frac{q}{q_0} = \frac{\theta_1'(0)}{[\theta_1'(0)]_0} \quad (19)$$

Curve 2 agrees with the results of the reports [1, 2]. Curve shows the change in the heat flow taking the Joule heat into account. The ratio  $q/q_0$  in this case is expressed by the formula

$$\frac{q}{q_0} = \frac{\theta_1(0) - \theta_2(0)}{[\theta_1(0) - \theta_2(0)]_0} \quad (20)$$

For simplicity in the computations it is assumed that

$$T_0 = T_w = \frac{a_0 v}{c_p}, \quad x = \left(\frac{v}{a_0}\right)^{\frac{1}{2}} \quad (20a)$$

As seen from the graph in Fig. 2 the liberation of Joule heat leads to an increase in the heat emission onto the wall as compared with the case where the Joule heating is disregarded. However, this increase is insignificant (of the order of 5%). The greatest reduction in heat flow attained with the value of the parameter  $S = 1.5$  amounts to 20%. To this correspond, for example, the following values of the parameters of flow and magnetic field:

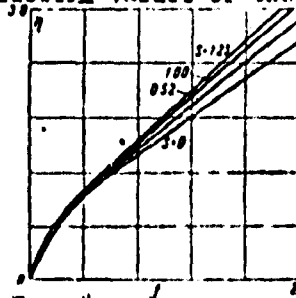


Fig. 4

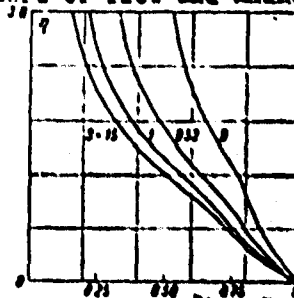


Fig. 5

(20b)  
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In this way the reduction in the heat transmission with the aid of a magnetic field for those parameters of gas which arise in the movement of a body at hypersonic speed in the lower layers of the atmosphere is not effective.

In Fig. 5 there are constructed the profiles of the temperature  $\theta =$

$(T - T_0) / (T_w - T_0)$  for different values of  $S$  and for

$$T_0 = T_w = \frac{a_0 v}{c_p}, \quad x = \left(\frac{v}{a_0}\right)^{\frac{1}{2}} \quad (20a)$$

Entered March 9, 1962

$$\alpha_0 \sim 5 \cdot 10^3 \text{ ссм}^{-1}, \quad \rho_0 \sim 1.6 \cdot 10^{-3} \text{ кг/м}^3, \quad \varepsilon \sim 300 \text{ мО/м}, \quad H_0 \sim 3800 \text{ эрстеда}$$

206

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# DETERMINING THE BASE PRESSURE AND BASE TEMPERATURE ON THE SUDDEN EXPANSION OF A SONIC OR SUPERSONIC FLOW

by

R. K. Tagirov

The article discusses the method of determining the base pressure and base temperature on the sudden expansion of a plane or axially symmetrical flow. The method is based on the known method by Korst [1], but in contrast to his method there is taken into account the non-isothermic quality of the mixing, and it is extended to cases of sudden expansion of an axially symmetrical flow toward the axis of symmetry.

There is presented a comparison of the results of the computation with the data of other authors.

**Sec. 1. Determining the Base Pressure** There is assumed the following system of flow with four characteristic regions (Fig. 1):

- region of flow to the region of sudden expansion (1);
- region of expansion of the flow in the Prandtl-Meyer wave (2);
- region of mixing of the flow bordering on the stagnant zone (3);
- region of increase in pressure in the end of the stagnant zone (4).

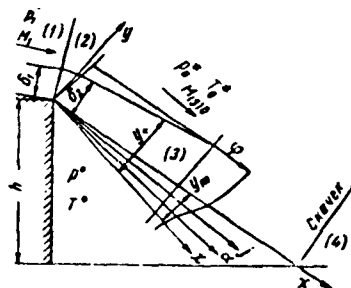


Fig. 1. (Скочек = jump)

In solving the problem one makes the following assumptions.

- 1) The static pressure is constant in the region (3) and equal to the pressure of the undisturbed core of the flow, i. e.,  $p^0 = p_{30}$ . Here and further on the superscript  $^0$  designates the base pressure and temperature, and the subscript  $0$  indicates

the parameters of the core of the flow.

2) The base temperature  $T^0$  is constant in the whole stagnant zone with the exception of a thin hot boundary layer at the walls.

3) In the zone of mixing  $\frac{T^* - T^0}{T_{0^*} - T^0} = Q \left( \varphi = \frac{U}{U_{30}} \right)$ . Here  $T^*$  is the temperature of retardation and  $Q$  is the factor of velocity representing the ratio of the velocity in the zone of mixing to the velocity of the core of the flow.

4) After the turn in the wave of expansion the profile of velocities of the boundary layer is described by an exponential law

$$\zeta_2^{\frac{1}{n}} = \varphi_2, \quad \zeta = y/\delta_2 \quad (1)$$

Here  $\zeta_2$  is the dimensionless ordinate and  $\delta_2$  is the thickness of the boundary layer behind the wave of expansion. It can be determined approximately from the equation of continuity.

Just as in the report [1] the problem is solved from a joint consideration of the flows of an elastic gas and an ideal gas, whereby by the latter one understands a fictitious flow which has the same values  $M_1$  and  $p^0/p_1$  as the flow of the viscous gas with unchanged geometry of the channel or the body around which the flow occurs.

On the boundary of this nonviscous flow there is introduced in the general case an orthogonal curvilinear system of coordinates  $XY$ , while the axis  $X$  is directed along the boundary line of flow. In the zone of mixing of the viscous gas there is introduced an analogous orthogonal curvilinear movable system of coordinates  $x, y$ , which according to the assumption (see [1]) is somewhat shifted in the direction of the axis  $Y$ , so that  $X \approx x$ ,  $Y = y - y_m(x)$  where  $y_m(0) = 0$ .

The approximation solution of the simplified equation of motion in the zone of mixing obtained in the report [1] is written, taking into consideration the assumption 4) made above, in the form:

$$\varphi = \frac{1}{2} [1 + \operatorname{erf}(\eta - \eta_p)] + \frac{1}{\sqrt{\pi}} \eta_p^{-\frac{1}{2}} \int_{\eta - \eta_p}^{\eta} (\eta - \beta)^{\frac{1}{2}} e^{-\beta^2} d\beta \quad (2)$$

Here the dimensionless coordinates

$$\eta_p = c \left( 2 \int_0^{\psi} \psi / (\psi) d\psi \right)^{-1/2}, \quad \eta = \zeta \eta_p, \quad \psi = \frac{x}{a_1} \quad (3)$$

On the basis of experimental data in Korst's work it is assumed

$$c = 12 + 2.578 M, \quad f(\psi) = 1 + a e^{-b\psi}, \quad b = \text{const} = 0.17 \quad (3a)$$

The value  $a$  is determined in accordance with the experimental dependence which can be approximated in the following form:

$$a = 0.334 \gamma^{-1.11} = 0.334, \quad \gamma = \left( 1 - \frac{\delta_2^*}{\delta_1^*} - \frac{\delta_2^{**}}{\delta_1^{**}} \right) / \left( 1 - \frac{\delta_2^*}{\delta_1^*} \right) \quad (4)$$

Here  $\delta_2^*$  and  $\delta_2^{**}$  are the thickness of the dislodgment and the thickness of the loss of the impulse in the boundary layer behind the wave of expansion.

We will assume that these empirical relationships and the profile of velocities are valid for the non-isothermic and the isothermic plane and axially symmetrical flows.

The position of the coordinate system  $X, Y$  can be determined if one knows the value  $p^0$ , since the boundary of the ideal flow can be constructed by the method of the characteristics or some other approximation process.

The position of the coordinate system  $x, y$  with relation to  $X, Y$  can be determined by one coordinate  $y_m$  with the aid of the equation of the amount of motion.

If one introduces Crocco's number  $C$  and the plus sign for the parameters determinable on the boundary line of the flow of the zone of mixing, which borders on the core of the flow, then for the plane flow the unknown relationship will have the form

$$\eta_m = \eta_+ - \eta_p + (1 - C_{20}) [\eta_p K_1 - J_{21}] \left( C = \left[ 1 + \frac{2}{(k-1)M^2} \right]^{-k} \right) \quad (5)$$

Here

$$K_1 = \int_0^1 \frac{\eta^{1/2} d\eta}{\psi + \eta(1-\psi) - \eta^2 C_{20}^2}, \quad J_{21} = \int_{-\infty}^{\eta} \frac{\eta^{1/2} d\eta}{\psi + \eta(1-\psi) - \eta^2 C_{20}^2} \quad (6)$$

$\psi = T^0/T_0^*$  is the dimensionless bottom temperature, and  $k$  is the adia-

batic curve.

The value  $\eta_+$  is determined from the condition

$$1 - \varphi(\varphi_1, \eta_p, \eta_+) < \epsilon, \quad (7)$$

where  $\epsilon$  is a small magnitude.

Here and from here on the integrals  $J$  will have a double subscript-- the first will indicate the number of the integral, and the second will indicate to what line of the flow the integral refers. For the axially symmetrical flow the magnitude  $\eta_m$  is set up in the form

$$\eta_m = -B_1 \pm \sqrt{B_1^2 + A_1} \quad (8)$$

Here

$$\begin{aligned} B_1 &= (1 - C_{30}^2) J_{2+} + \frac{\tau_2 \eta_p}{\cos \psi_2} - \eta_+ \\ A_1 &= \pm \frac{2\tau_1 \eta_+ \eta_p}{\cos \psi_1} - \eta_+^2 \mp (2\tau_1 \mp \cos \psi_1) \eta_p^2 \frac{1}{\cos \psi_2} \mp \\ &\quad \mp \frac{2\tau_1 \eta_p (1 - C_{30}^2) J_{2+}}{\cos \psi_2} + 2(1 - C_{30}^2) J_{2+} \pm \\ &\quad \pm 2\tau_1 \eta_p^2 (1 - C_{30}^2) \frac{K_2}{\cos \psi_2} - 2 \frac{\cos \psi_1}{\cos \psi_2} (1 - C_{30}^2) \eta_p^2 K_3 \\ K_2 &= \int_0^1 \frac{\varphi_1^2 \zeta d\zeta}{\theta + \varphi_2(1 - \theta) - \varphi_1^2 C_{30}^2}, \quad J_2 = \int_{-\infty}^{\infty} \frac{\varphi^2 \eta d\eta}{\theta + \varphi(1 - \theta) - \varphi^2 C_{30}^2} \end{aligned} \quad (9)$$

By  $\psi_1$  and  $\psi_2$  there are indicated the angles of inclination of the vectors of velocity on the boundary of the ideal flow, respectively, for the section which is immediately behind the wave of expansion, and for the section where the jump in density occurs. It is assumed that in the zone of mixing of the viscous flow the angles of inclination of the vectors of velocity in the respective section will be the same  $\tau_1 = r_1/\delta_2$  and  $\tau_2 = r_2/\delta_2$ , respectively, the dimensionless radii of the boundary of the ideal flow for the two sections indicated above. It is easy to establish that if  $\eta_p = 0$ , then the corresponding expressions for  $B_1$  and  $A_1$  are obtained by the replacing in the above-written equations of the values  $\eta_p/\delta_2$  by  $\delta/x$ . Such a transition will be valid also for all the relationships following below. Let us note that here and further on the upper signs are real for the case of expansion of

the axially symmetrical flow from the axis, and the lower ones toward the axis. Further, as in the report [1] there are introduced into the consideration two characteristic lines of flow in the zone of mixing: the line of constant mass  $j$  and the demarcation line of flow  $d$ .

The use of the equation of continuity enables one to determine the coordinate of the line of flow  $j$ .

For the plane flow this relationship has the form

$$J_{1+} = J_{1j} = J_{2+} + (K_1 - K_2) \eta_p \quad (10)$$

where

$$J_1 = \int_{-\infty}^{\eta} \frac{\varphi \eta d\eta}{\vartheta + \varphi(1-\vartheta) - \varphi^2 C_{\infty}^2}, \quad K_1 = \int_0^1 \frac{\varphi_2^2 d\zeta}{\vartheta + \varphi_2(1-\vartheta) - \varphi_2^2 C_{\infty}^2} \quad (11)$$

If into the stagnant zone there enter a supplementary mass of fluid then this mass should be removed between the lines of flow  $j$  and  $d$ , since in the stagnant zone there should be maintained a constancy of mass

$$G_d + G_b = 0 \quad (1.1)$$

where  $G_b$  is the mass of the gas entering into the stagnant zone. The expression for the mass of gas removed between the lines  $j$  and  $d$  has the form

$$G_d = - \frac{\delta_2 p^0 k M_{\infty}}{\sqrt{k g R T_0^*} \tau(M_{\infty})} \frac{1 - C_{\infty}^2}{\eta_p} (J_{1d} - J_{1j}) \quad (1.1a)$$

where  $R$  is the gas constant;  $\tau(M_{\infty})$  is the gas-dynamic function.

Analogous equations can be written for the case of axially symmetrical flow.

The equation for determining  $\eta_j$

$$(\tau_2 \eta_p \pm \eta_m \cos \psi_2) (J_{1+} - J_{1j}) \mp \cos \psi_2 (J_{4+} - J_{4j}) = N_1 \quad (1.1b)$$

where

$$N_1 = \tau_1 \eta_p^2 K_1 \mp \eta_p^2 \cos \psi_1 K_4 + (\tau_2 \eta_p \pm \eta_m \cos \psi_2) J_{2+} \mp \cos \psi_2 J_{2+} - \tau_1 \eta_p^2 K_2 \pm \eta_p^2 \cos \psi_1 K_3 \quad (1.1c)$$

$$J_4 = \int_{-\infty}^{\eta} \frac{\varphi \eta d\eta}{\vartheta + \varphi(1-\vartheta) - \varphi^2 C_{\infty}^2}, \quad K_4 = \int_0^1 \frac{\varphi_2^2 d\zeta}{\vartheta + \varphi_2(1-\vartheta) - \varphi_2^2 C_{\infty}^2} \quad (1.1c)$$

The expression for the mass removed between the lines  $j$  and  $d$

$$-G_d = \frac{\delta_2 p^0 k M_{\infty}}{\sqrt{k g R T_0^*} \tau(M_{\infty})} \frac{2\pi(1 - C_{\infty}^2)}{\eta_p^2} [(\tau_2 \eta_p \pm \eta_m \cos \psi_2) \times (J_{1j} - J_{1d}) \mp \cos \psi_2 (J_{4j} - J_{4d})] \quad (1.1d)$$

In the end of the stagnant zone there occurs an increase in pressure in an oblique jump in density. It is assumed that the intensity of this jump is determined by the parameters of an ideal flow  $p_4/p^0 = f(M_{30}, \psi_2)$ .

If the boundary of the ideal flow is established then the magnitude  $\psi_2$  will be determined if one knows the section where the jump in density occurs. i. e., if one knows  $x$  or  $\eta_p(\psi)$  which characterize the distance from the beginning of the coordinates to the jump.

For the case of the plane flow the angle  $\psi_2$  will be equal to the angle of deviation of the flow in the wave of expansion in zone (2).

For the case of sudden expansion of the axially symmetrical flow from the axis the position of the jump is determined from the intersection of the boundary of the ideal flow with the wall of the channel  $[1]$ .

If, however, the sudden expansion of the axially symmetrical flow occurs towards the axis, then the foregoing condition proves to be inapplicable, and the position of the jump is approximately determined from the intersection of the zero line of the flow (lower boundary of the zone of mixing) with the axis of symmetry.

This condition can be written in the form

$$\eta_m - \eta_{min} = \eta_p \frac{\tau_2}{\cos \psi_2} \quad (1.1e)$$

where  $\eta_{min}$  is the coordinate of the zero line of the flow, which is determined from the condition  $\varphi(\eta_2, \eta_p, \eta_{min}) < t_c$ . Let us note that with such an assumption the thickness of the zone of mixing can be easily determined

$$\frac{t}{h} = (\eta_2 - \eta_{min}) \frac{\cos \psi_2}{\eta_p} \frac{\delta_2}{h} \quad (h = \text{height of offset}) \quad (1.1f)$$

If, however, around the plane or ring-shaped offset there flow to different currents, then the magnitude of the pressure behind the jump in density is determined from the condition of equality of pressures of the inner and outer flows  $p_4^+ = p_4^- = p_4$ .

Here and from here on the superscript + refers to the inner flow and the superscript — to the outer.

As has already been mentioned above in the stagnant zone there should be fulfilled the condition of preservation of the mass. This condition, which enables one to obtain a solution of the problem, was proposed and successfully applied in the report [1].

If one assumes that on the line of flow d the level of mechanical energy is determined by the pressure of retardation  $p_{3d}$ , then one may say that with a full conversion of the kinetic energy as a result of the retardation of the particles in the region (4) there is obtained the static pressure  $p_4$ , i. e.,

$$\frac{p_{4d}^+}{p^+} = \frac{p_4}{p^+} \quad (1.2)$$

This also serves to be the closing condition for the case of sudden expansion of one flow.

From the condition of adiabatic retardation in a given point it is easy to find

$$\frac{p_{4d}^+}{p^+} = \left[ 1 - \frac{\varphi_d^2 C_{\infty}^2}{\varphi + \varphi_d(1 - \varphi)} \right]^{-\frac{k}{k-1}} \quad (1.2a)$$

If the stagnant zone is open, i. e., if there flows into it (or from it) a given mass of fluid  $G_b$ , then the coordinate of the demarcation line of the flow  $\eta_d$  can be determined from the condition (1.1). If the stagnant zone is open, i. e.,  $G_b = 0$ ,  $G_d = 0$ , then  $\eta_d = \eta_j$ .

Let us now consider briefly what form the problem closing condition will have in the case of two different currents flowing around a plane or ring-shaped offset. In this case clearly one should consider the model of an open stagnant zone [2]. The coordinate  $\eta_d$  of each flow is determined from the condition (1.2).

The closing condition here will be the condition of constant mass in the stagnant zone

$$G_d^+ + G_d^- + G_b = 0 \quad (1.2b)$$

The expressions for  $G_d$  were given above.

In this way one solves the problem of determining  $p^0$  if one knows the magnitude  $T^0$ .

Sec. 2. Determining the Base Temperature  $T^0$  We will consider, as usual, that all the assumptions are valid which are made above in Sec. 1. Initially let us determine the specific heat flow  $q$ .

By assuming that the mechanism of the turbulent exchange for the impulses and the heat are identical (see [3]) one may write at once

$$q = c_p g \rho \varepsilon \frac{\partial T}{\partial y} \quad (1.2c)$$

Here  $c_p$  is the heat capacity,  $\rho$  is the density, and  $g$  is the acceleration of the force of gravity.

The coefficient of turbulent viscosity  $\varepsilon$  is determined from the expression which was used in the report [1] in determining the profile of the velocities  $\varphi$

$$\varepsilon = \frac{1}{2\sigma^2} \psi \delta_2 u_{22} / (\psi) \quad (1.2d)$$

With the use of the relationships of Sec. 1 after some transformations, we get the expression for the specific heat flow through a single area of the surface the generatrix of which is the zero line of the flow ( $\varphi = 0$ )

$$q = \frac{c_p}{R} \frac{1}{2\sigma^2} \psi (1 + a\tau^{-b}) \eta_p(\psi) p^0 M_{30} \sqrt{kgRT_0^* \tau(M_{30})} \frac{1-\delta}{\delta} \left( \frac{\partial \varphi}{\partial \eta} \right)_{\eta=0} \quad (1.2e)$$

For the line of flow where  $\varphi \approx 0$  the dimensionless coordinate will have a determined value  $\eta \approx \eta_{\min}$ , which was discussed above. The magnitude of the unknown  $T^0$  can be established from the equation of the heat balance. For the case of sudden expansion of the axially symmetrical or plane flow

this equation can be written in the form

$$N \frac{T_0^* - T^0}{T^0} = AT^0 + B \quad (1.2f)$$

Here

$$N = \delta_2 (2\pi r_m)^2 \frac{c_p}{R} \frac{1}{2\sigma^2} p^0 M_{30} \sqrt{kgRT_0^* \tau(M_{30})}$$

$$B = \int_0^{\psi} \psi (1 + a\tau^{-b}) \eta_p(\psi) \left( \frac{\partial \varphi}{\partial \eta} \right)_{\eta=0} d\psi \quad (1.2g)$$

whereby for the plane flow  $i = 0$ , and for the axially symmetrical flow  $i = 1$ . The average radius of the zero line of flow is indicated by  $r_m$ .

In the framework of this study it is considered that A and B are known magnitudes determinable by the relationship  $Q^+ + Q^- = AT^0 + B$  where  $Q^+$  is the heat passed with the blown gas, and  $Q^-$  is the heat passing through the wall into the stagnant zone.

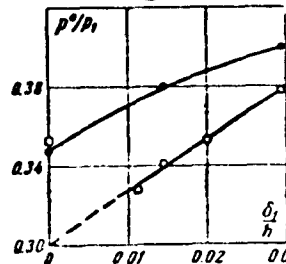


Fig. 2.  $k = 1.4$ ,  $M = 2.025$ ,  $\bar{v} = 1$ .

○ is the experiment [5], ● is the computation, and □ is the computation [1].

It is necessary to stipulate that, although in principle the magnitudes A and B can be determined, to find them practically can involve some difficulty, for example, such as is connected with the establishing of coefficients of heat emission on the wall in the stagnant zone. They may, however, be approximately determined with the aid of the report [4]. But these questions go beyond the limit of the present study, and therefore they are not being considered here.

In the case of flowing around a flat or ring-shaped offset of two different streams the problem of the heat balance can be written in the form

$$-N \frac{T_0^{**} - T^0}{T^0} F^- = N^+ \frac{T_0^{**} - T^0}{T^0} F^+ + AT^0 + B \quad (1.2h)$$

For the case of  $\eta_p = 0$  the magnitude  $T^0$  in case of sudden of one flow is determined from the relationship

$$\Pi \frac{T_0^{**} - T^0}{T^0} = AT^0 + B \quad (1.2i)$$

Here

$$\Pi = (2\pi r_m)^i x \frac{c_p}{2H\sqrt{\pi}} \exp(-\eta_{\min}^2) \frac{P^0}{\sigma} M_{30} \sqrt{kgRT_0^{**} \tau(M_{30})}. \quad (1.2j)$$

If  $\eta_p = 0$  and two different streams are flowing around an offset the magnitude  $T^0$  is determined from the equation

$$-\Pi \frac{T_0^{**} - T^0}{T^0} = \Pi^+ \frac{T_0^{**} - T^0}{T^0} + AT^0 + B \quad (1.2k)$$

If one considers

$$A \approx 0, \quad B \approx 0, \quad r_m^{i+} x^- \approx r_m^{i+} x^+ \quad (1.21)$$

and disregards the difference in the physical characteristics of the two streams it is possible to obtain a very simple formula for determining the bottom temperature

$$\theta = \frac{T_w}{T_0} = \left( \frac{T_w}{T_0} + \frac{\lambda_{30}}{\lambda_{30}^+} \sqrt{\frac{T_w}{T_0}} \right) / \left( 1 + \frac{\lambda_{30}}{\lambda_{30}^+} \sqrt{\frac{T_w}{T_0}} \right) \quad (1.22)$$

Here  $\lambda_{30}^+$  and  $\lambda_{30}^-$  are the coefficients of the velocity for the undisturbed core of the respective flows.

### Sec. 3. Results of Computations and Comparison with Data of Other

Authors The results of the computation of the base pressure  $p^0$  in the case of an offset around which is flowing a horizontal stream, having  $M_1 = 2.025$  and  $k = 1.4$ , with different values for  $\delta_1/h$ , are presented in Fig. 2. Their comparison with the experimental data of the report [5] shows that

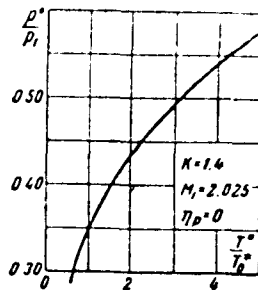


Fig. 3

computation by the method expounded as well as the method of the work [1] gives somewhat higher results, but from the viewpoint of practical application, the agreement can be considered as satisfactory.

In Fig. 3 there are presented the results of the computation of the dependence of  $p^0/p_1$  on  $U^0/U_1$  where around an offset there is flowing a horizontal stream having  $M_1 = 2.025$ ,  $k = 1.4$ , and  $\eta_p = 0$ .

In the computation there was used a constant approximated value for the coefficient of heat emission through the wall in the stagnant zone. The result obtained corresponds qualitatively with the experimental data of the report [6] and with the computed data of the report [7].

The computations of  $p^0$  presented for the case of the sudden expansion

of the axially symmetrical flow from the axis showed that in the first place they find themselves in agreement with the experimental and computation data of other authors, and in the second place the computations can be made with the use of the formulas obtained for the horizontal flow.

Let us dwell now a little more in detail on the results of the computation of  $p^0$  in the case of sudden expansion of the axially symmetrical flow towards the axis.

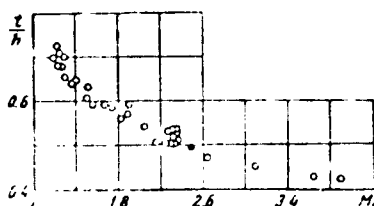


Fig. 4.  $k = 1.4$ .  $\circ$  is the experiment [8],  $\bullet$  is the computation,  $\eta_D = 0$ ,  $\psi = 1$ .

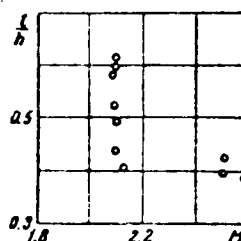


Fig. 5.  $k = 1.4$ .  $\circ$  is the experiment,  $\bullet$  is the computation,  $\eta_D = 0$ ,  $\psi = 1$ .

In Fig. 4 there are presented the results of the computation of the magnitude  $t/h$  representing the thickness of the zone of mixing in the section where the jump occurs, and for comparison there are presented the experimental data of the report [8], obtained with the flow around a missile. In Fig. 5 there are presented the computation point  $t/h$  and the experimental data of the author of this work, obtained by the processing of shadow pictures of a flow in a channel as depends on the  $M_{geom}$  number determined by ratio of the area of the channel cross section  $F_1/F_2$ .

Comparison shows that both in the case of external flow-around, and in the case of sudden expansion in the channel towards the axis, the agreement between the computation and experimental magnitudes of  $t/h$  can be considered as quite satisfactory. This points to the fact that apparently the method

proposed above for determining the location of the jump in density for the case of sudden expansion of the axially symmetrical flow towards the axis, proves to be sufficiently acceptable, at least for making engineering calculations.

The results of the computations of  $p^0$  for these same cases considered above are given in Figs. 6 and 7. Comparison with the corresponding experimental data shows their agreement is fully satisfactory.

Here one should note that the computations made with the use of the

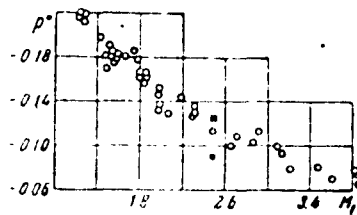


Fig. 6.  $p^0 = \frac{2}{\kappa M_1} \left( \frac{p^0}{p_1} - 1 \right)$ .

$\kappa = 1.4$ .  $\circ$  is the experiment [8].

Computation of  $\eta_p = 0$ ,  $\vartheta = 1$ ,  $\bullet$  is

with the use of the formula of the

horizontal flow,  $\blacksquare$  is with the use of

formula of the axially symmetrical flow

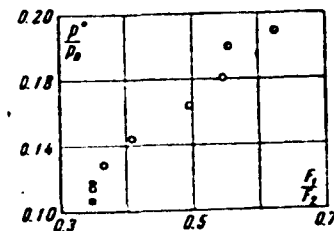


Fig. 7.  $\kappa = 1.4$ ,  $M_1 = 0$ .  $\circ$  is

the experiment. Computation of  $\eta_p =$

0,  $\vartheta = 1$ .  $\bullet$  is with the use of the

of the horizontal flow,  $\blacksquare$  is with

the use of the formula of the axially

symmetrical flow.

formulas for the horizontal flow and for the axially symmetrical flow give noticeably different results, and therefore in such cases where the expansion of the axially symmetrical flow proceeds to the axis of symmetry, the computations, in all probability, must be made with the use of the formulas obtained for the axially symmetrical flow.

In conclusion let us note that all the computations were made with the assumption that  $n = 7$  and  $\eta_{\min} = -2$  with the use of the tables of the auxiliary integrals  $J$ ,  $K$ ,  $F$ , and  $\varphi$  obtained by computation on the electron com-

puting machine.

Entered April 11, 1961

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